647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

December - 2020 (Old Course)

MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Que-1 (a) Answers the following. (any two) (10)

1. If (X , d) is a metric space, then show that $d_1 : X \times X \to \mathbb{R}$; $d_1(x, y) = \frac{d(x,y)}{1+ d(x,y)}$ is a bounded metric on X.
2. Let X be a metric space and $A, B \subset X$. Show that $\underline{A}$____ $\cup \bar{B}$.
3. Prove that if (X , d) is a metric space, then $|d(x, z) - d(y, z)| \leq d(x, y)$, $\forall x, y, z \in X$.

(b) Attempt any one. (04)

1. Prove that : Any finite subset of a metric space is closed.
2. Define following terms :

(i) Discrete metric space (iii) Open set

(ii) Limit point of a set (iv) Dense Set

Que-2 (a) Answers the following. (any two) (10)

1. S
h
o
w
2. Show that : $f(x) = 2 - 3x$ is integrable on $[1, 3]$ and $\int_1^3 (2-3x)\, dx = -8$.
3. Show that : If f and g are bounded and integrable on $[a, b]$ then $f\,g$ is also integrable on $[a, b]$.

(b) Attempt any one. (04)

1. I
f
2. If $f \in R[a, b]$, then show that
f
$m(b\text{-}a) \leq \int_a^b f(x)\, dx \leq M(b\text{-}a)$.
where m and M are the infimum and supermum of f on $[a, b]$, respectively.
.

Que-3 (a) Answers the following. (any two) (10)

1. State and Prove : First Mean Value Theorem of Integral Calculus.
2. Show that $GL(2, \mathbb{R})$ is a non-abelian group under matrix multiplication.
3. Let G be a group. For $a, b \in G$ and $n \in \mathbb{Z}$, prove that $(aba^{-1})^n = ab^na^{-1}$.

(b) Attempt any one. (04)

1. E x p r

2. Let $G = \{f_{a,b} \mid f_{a,b} : \mathbb{R} \to \mathbb{R}, f_{a,b}(x) = ax + b \ (x \in \mathbb{R}), a \neq 0, a, b \in \mathbb{R}\}$ be a group under composition of mappings. Find the identity element of G and inverse of $f_{2,3}$.

Que-4 (a) Answers the following. (any two) (10)

1. Let S_n be the group of permutations on n-symbols and A_n be the set of all even permutations in S_n. Then show that A_n is a normal subgroup of S_n.
2. State and prove Lagrange's Theorem.
3. If G is a group such that $(ab)^n = a^n b^n$, for three consecutive integers n, then show that G is abelian.

(b) Attempt any one. (04)

1. Let G be a group. If $a^2 = e$, $\forall_a \in G$, then show that G is abelian.
2. Let G be a group and $H \leq G$. For $a, b, \in G$ define $a \equiv b \pmod{H} \Leftrightarrow a^{-1}b \in H$. Prove that $\equiv$ is an equivalence relation on G. Also find $cl(a)$ where $a \in G$.

Que-5 (a) Answer the following. (any two) (10)

1. State and prove Cayley's Theorem.
2. Let $\phi : G \to \bar{G}$ be a group isomorphism. Then show that $\phi(a^n) = \phi(a^n)$ for $n \in \mathbb{Z}$.
3. Let $f : G \to \bar{G}$ be a group isomorphism. If $\overline{N} \lhd \bar{G}$, then show that $f^{-1}(\overline{N}) \lhd G$.

(b) Attempt any one. (04)

1. Let H and K be finite sub groups of a group G. if $O(H)$ and $O(K)$ are relatively prime, then prove that $H \cap K = \{e\}$.
2. Prove that a group of prime order is cyclic.

****